Equations with Two Variables

Sometimes, you will want to write equations that have more than one variable. For example, you may order a dozen doughnuts two days in a row. The first day, you get 8 maple bars and 4 chocolate glazed for $10. The second day, you get 4 maple bars and 8 chocolate glazed for $8. Since the shop doesn’t print their prices on the menu, how can you find out how much a maple bar costs?

Let **x** stand for a maple bar and **y** stand for a chocolate glazed. First, set up the equations:

 8x + 4y = 10

 4x + 8y = 8

Now, let’s look at a couple of ways to solve for x and y.

Substitution

Pick one of the equations and solve for either x or y. Let’s solve for x in the second equation:

 4x + 8y = 8

 4x = 8 - 8y *Subtracting 8y from each side*

 x = 2 - 2y *Dividing both sides by 4*

Now, let’s substitute (2 - 2y) for x in the first equation

 8(2 - 2y) + 4y = 10

 16 - 16y + 4y = 10 *Using the distributive property*

 16 - 12y = 10 *Combining like terms*

 -12y = -6 *Subtracting 16 from both sides*

 y = .5 *Dividing both sides by -6*

Going back to the original equation, now that we know y = .5

 8x + 4(.5) = 10 *Substituting .5 for y*

 8x + 2 = 10

 8x = 8 *Subtracting 2 from both sides*

 x = 1 *Dividing both sides by 8*

Adding Equations Together

Another way to solve systems of equations is to add them together to eliminate one of the variables. Looking at our two equations:

 8x + 4y = 10

 4x + 8y = 8

Notice that the first equation has twice as many x as the 2nd. Let’s divide the first equation by 2:

 4x + 2y = 5 *Dividing both sides by 2*

Now the two equations are:

 4x + 2y = 5

 4x + 8y = 8

Now that both equations contain the term 4x, let’s multiply the first equation by -1.

 -4x -2y = -5 *Multiplying both sides by -1*

Adding the two equations together:

 -4x -2y = -5

+ 4x + 8y = 8

 0x + 6y = 3 *Notice that x is eliminated because -4x + 4x = 0x = 0*

Solving for y:

 6y = 3

 y = .5 *Dividing both sides by 6*

Again, going back to one of the original equations, now that we know y = .5

 4x + 8(.5) = 8 *Substituting .5 for y*

 4x + 4 = 8

 4x = 4 *Subtracting 4 from both sides*

 x = 1 *Dividing both sides by 4*