Exponents

An *exponent* tells how many times a number is to be multiplied by itself. For example:

24 = 2 x 2 x 2 x 2 = 16.

4 is the *exponent* or the power of 2. We say that 2 is raised to the 4th power, or call it simply, “2 to the fourth.” *Important: Do not confuse 24 with 2 x 4. THEY ARE NOT THE SAME. 2 x 4 = 8 and 24 = 16.*

Squares and Cubes

When we raise a number to the second power, we call it a square. Here’s why:



🡨 The area of this square is 5 x 5 or 52.

When a number is raised to the third power, it’s called a cube.



🡨 The volume of this cube is 5 x 5 x 5 or 53

52 and 53 are usually read as “five squared” and “five cubed”.

Square and Cube Roots

The opposite of a square is a *square root* and the opposite of a cube is a *cube root*. The symbol for a square root is $\sqrt{ }$ or$\sqrt[2]{ }$. The symbol for a cube root is $\sqrt[3]{ }. $

So if 52 = 25, then $\sqrt{25 }$ = 5

And if 53 = 125, then $\sqrt[3]{125 }$ = 5

These would be read as, “The square root of 25 is 5 and the cube root of 125 is 5.”

Similarly, $\sqrt{x^{2}}$ = x and $\sqrt[3]{x^{3}}$ = x, or, “The square root of x squared is x and the cube root of x cubed is x.”

Powers of 1 and 0

Any number to the power of 1 is that number. For example:

21 = 2 and 3,4321 = 3,432

Any number to the power of 0 is 1. For example:

20 = 1 and 314,1590 = 1

Powers of 10 and Scientific Notation

We write 10 x 10 x 10 = 1000 as 103. Powers of 10 are very easy to work with because all you do is count the zeroes.

4 x 105 = 4 x 100,000 = 400,000

6 x 108 = 6 x 100,000,000 = 600,000,000

Scientists must work with very large numbers. For example, the distance from the sun to the planet Neptune is about 2,790,000,000 miles. We can express this as:

2.79 x 1,000,000,000 or

2.79 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 or

2.79 x 109

These numbers are easier to write and compute when expressed as scientific notation. In scientific notation, the number is expressed as a number between 1 and 10 and then multiplied by a power of 10. The number of the exponent is the number of zeros added on the end. If there is a decimal point, you have to move the decimal point to the right, counting for each, and then add zero's on the end to finish off. For example:

2.012 x 103 = 2012 and 1492 = 1.492 x 103

Multiplying and Dividing Exponents

As long as they have the same base, multiplying and dividing numbers with exponents is as easy as 1st grade addition and subtraction. For example:

25 x 24 = 2(5+4) = 29  and 25 ÷ 24 = 2(5-4) = 21 = 2

Does it seem too easy? Here’s why it works:

25 x 24 = (2 x 2 x 2 x 2 x 2) x (2 x 2 x 2 x 2) = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 = 29

25 ÷ 24 = $\frac{2 x 2 x 2 x 2 x 2}{2 x 2 x 2 x 2}$ = $\frac{2 x 2 x 2 x 2 x 2}{2 x 2 x 2 x 2}$ = $\frac{2}{1}$ = 2

Order of Operations

Since at least the 1500s, mathematicians have followed the rules of precedence called the “order of operations.” Given a long or complicated equation, these rules tell you how to decide which of the operations in the equation (addition, subtraction, multiplication, division, exponents & grouping) to carry out in which order.

A common technique for remembering the order of operations is the abbreviation "PEMDAS", which is turned into the phrase "Please Excuse My Dear Aunt Sally". It stands for "Parentheses, Exponents, Multiplication and Division, and Addition and Subtraction". This tells you the ranks of the operations: Parentheses outrank exponents, which outrank multiplication and division (but multiplication and division are at the *same* rank), and these two outrank addition and subtraction (which are together on the bottom rank). When you have a bunch of operations of the same rank, you just operate from left to right. For instance, 15 ÷ 3 × 4 is not 15 ÷ 12, but is rather 5 × 4, because, going from left to right, you get to the division first. Some examples:

**Simplify 4 – 3[4 –2(6 – 3)] ÷ 2**

Simplify from the inside out: first the parentheses, then the square brackets, being careful to remember that the "minus" sign on the 3 in front of the brackets goes with the 3. Only once the grouping parts are done will you do the division, followed by adding in the 4.

4 – 3[4 –2(6 – 3)] ÷ 2
    = 4 – 3[4 – 2(3)] ÷ 2
    = 4 – 3[4 – 6] ÷ 2
    = 4 – 3[–2] ÷ 2
    = 4 + 6 ÷ 2
    = 4 + 3
    = **7**

**Simplify 16 – 3(8 – 3)2 ÷ 5**

Remember to simplify inside the parentheses *before* you square, because (8 – 3)2 is *not* the same as 82 – 32.

16 – 3(8 – 3)2 ÷ 5
    = 16 – 3(5)2 ÷ 5
    = 16 – 3(25) ÷ 5
    = 16 – 75 ÷ 5
    = 16 – 15
    = **1**

Non-Base 10 Number Systems

We use a base-10 number system, most likely because we have ten fingers. If we only had four fingers on each hand, we would probably be using a base-8 system. When we talk about numbers in different bases, we use the following notation:

**3148** = “314 base-8” and **10010012** = “1001001 base-2”

In any number system, we can talk about the place value of a digit in a number. For example, in base-10:

2,01310 = (2 x 103) + (0 x 102) + (1 x 101) + (3 x 100) or

 (2 x 1000) + (0 x 100) + (1 x 10) + (3 x 1)

We say that 2 is in the thousands place, 0 is in the hundreds place, 1 is in the tens place and 3 is in the ones place.

In base-4, the same number (2,013) would be written like this:

2,0134 = (2 x 43) + (0 x 42) + (1 x 41) + (3 x 40) or

 (2 x 64) + (2 x 16) + (1 x 4) + (3 x 1)

We say that 2 is in the sixty-fours place, 2 is in the sixteens place, 1 is in the fours place and 3 is in the ones place.

Binary Numbers (Base 2)

Let's look at base-two, or binary, numbers. How would you write, for instance, 1210 ("twelve, base ten") as a binary number? In base ten, you have columns or "places" for 100 = 1, 101 = 10, 102 = 100, 103 = 1000, and so forth. Similarly in base two, you have columns or "places" for 20 = 1, 21 = 2, 22 = 4, 23 = 8, 24= 16, and so forth.

The first column in base-two math is the units column. But only "0" or "1" can go in the units column. When you get to "two", you find that there is no single solitary digit that stands for "two" in base-two math. Instead, you put a "1" in the twos column and a "0" in the units column, indicating "1 two and 0 ones". The base-ten "two" (210) is written in binary as 102.

A "three" in base two is actually "1 two and 1 one", so it is written as 112. "Four" is actually two-times-two, so we zero out the twos column and the units column, and put a "1" in the fours column; 410 is written in binary form as 1002. Here is a listing of the first few numbers:

|  |  |  |
| --- | --- | --- |
| **Decimal**(base 10) | **Binary**(base 2) |  |
| 012345678910111213141516 | 0110111001011101111000100110101011110011011110111110000 | 0 ones1 one1 two and zero ones1 two and 1 one1 four, 0 twos, and 0 ones1 four, 0 twos, and 1 one1 four, 1 two, and 0 ones1 four, 1 two, and 1 one1 eight, 0 fours, 0 twos, and 0 ones1 eight, 0 fours, 0 twos, and 1 one1 eight, 0 fours, 1 two, and 0 ones1 eight, 0 fours, 1 two, and 1 one1 eight, 1 four, 0 twos, and 0 ones1 eight, 1 four, 0 twos, and 1 one1 eight, 1 four, 1 two, and 0 ones1 eight, 1 four, 1 two, and 1 one1 sixteen, 0 eights, 0 fours, 0 twos, and 0 ones |

Converting Binary Numbers to Base 10

Converting between binary and decimal numbers is fairly simple, as long as you remember that each digit in the binary number represents a power of two. Example:

**Convert 1011001012 to the corresponding base-ten number.**

First list the digits in order, and count them off from the RIGHT, starting with zero:

|  |  |
| --- | --- |
| digits:   | 1  0   1  1  0  0  1  0  1 |
| numbering:   | 8  7   6  5  4  3  2  1  0 |

The first row above (labelled "digits") contains the digits from the binary number; the second row (labelled " numbering") contains the power of 2 (the base) corresponding to each digits. You can use this listing to convert each digit to the power of two that it represents:

(1 × 28) +(0 × 27) + (1 × 26) + (1 × 25) + (0 × 24) + (0 × 23) + (1 × 22) + (0 × 21) + (1 × 20)

    = (1 × 256) + (0 × 128) + (1 × 64) + (1 × 32) + (0 × 16) + (0 × 8) + (1 × 4) + (0 × 2) + (1 × 1)

    = 256 + 64 + 32 + 4 + 1

    = 357   Copyright © Elizabeth 2001-2011 All Rights Reserved

Thus **1011001012 converts to 35710**

Converting Base Ten Numbers to Binary Numbers

Converting decimal numbers to binaries is nearly as simple: just divide by 2.

**Convert 35710 to the corresponding binary number.**

To do this conversion, divide repeatedly by 2, keeping track of the remainders as you go.



These remainders tell you what the binary number is. Simply read the numbers from around the outside of the division, starting on top and wrapping your way around and down the right-hand side. As you can see:

**35710 converts to 1011001012.**

Converting Between Base 10 and Other Number Systems

The methods of conversions between base ten and base two will work for converting to and from any non-decimal base. Just remember that whatever base you’re working with will involve powers of that base. So while Base-10 numbers will have a ones column, a tens column and a hundreds column, and so on, Base-8 number will have a ones column, an eights column, a sixty-fours column and so on.