Prime Numbers

A prime is a counting number greater than 1 that is divisible only by itself and by 1.

Let’s find all of the prime numbers less than 100.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

1. Cross off the number 1. The number 1 is called a unit. It is neither prime nor composite and is a factor of all counting numbers.
2. Circle the number 2, and then cross off all other multiples of 2.
3. Circle the number 3, and then cross off all other multiples of 3.
4. Circle the number 5, and then cross off all other multiples of 5.
5. Circle the number 7, and then cross off all other multiples of 7.
6. Circle all of the remaining numbers—these are all of the prime numbers < 100!

Rules of Divisibility

1. All numbers are divisible by 1.
2. If a number is even, it is divisible by 2.
3. If the sum of all the digits in a number is divisible by 3, the number is also divisible by 3.
4. If the last two digits of a number are 00 or are divisible by 4, the number is also divisible by 4.
5. If the last digit of a number is 0 or 5, the number is divisible by 5.
6. If the number is divisible by 2 and 3, the number is also divisible by 6.
7. Take the last digit in the number. Double it, then subtract the result from the remaining digits. If the answer is 0 or divisible by 7, the whole number is divisible by 7. With larger numbers, this process can be repeated as necessary. Example—determine if 2016 is divisible by 7:

6 x 2 = 12. 201 – 12 = 189. 189 is divisible by 7, but if you didn’t know that, you could double the final digit of 189 (9 x 2 = 18) then subtract it from the remaining digits (18 – 18 = 0). Therefore, 2016 is divisible by 7.

1. If the last 3 digits are divisible by 8, the whole number is divisible by 8.
2. If the sum of all the digits of a number is divisible by 9, the number is also divisible by 9.
3. If the last digit of a number is 0, the number is divisible by 10.
4. If the sum of every 2nd digit minus the sum of all the other digits equals 11 or 0, the number is divisible by 11. Example—determine if 3729 is divisible by 11:

(7 + 9) – (3 + 2) = 16 – 5 = 11. Therefore, 3729 is divisible by 11.

Factors

A factor of a number N is a number which divides N exactly.

For example, the factors of 12 are 1, 2, 3, 4, 6 and 12. Notice that every number has itself and 1 among its factors. If a number has only itself and 1 as factors, that number is prime.

A number is factored completely when it is expressed as a product of primes.

For example, 72 = 2 x 2 x 2 x 3 x 3. 2 x 2 x 2 x 3 x 3 is the complete factorization (or prime factorization) of 72.

**Cool trick: How can you figure out how many factors a number has?**

1. Factor the number completely. Example: 72 = 2x2x2x3x3.
2. List the different prime factors (2 & 3) and count how many of each there are (**3** “2”s and **2** “3”s)
3. Add 1 to each count: 3 + 1 = 4 and 2 + 1 = 3
4. Multiply the results: 4 x 3 = 12
5. The number 72 has 12 factors! (1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72)

Greatest Common Factor

The Greatest Common Factor (GCF) of two numbers is the largest number that divides both of the numbers exactly. Some strategies for finding the GCF:

1. List all the factors of each number, compare the lists and find the largest common factor. Example--finding the greatest common factor of 24 and 30:

Factors of 24: **1**, **2**, **3**, 4, **6**, 8, 12, 24

Factors of 30: **1**, **2**, **3**, 5, **6**, 10, 15, 30

The largest number both lists have in common is 6, therefore the GCF of 24 & 30 is 6.

1. Test the factors of the smaller number in order from largest to smallest to see which is the first that divides evenly into the larger number. Example—finding the GCF of 16 and 56:

The factors of 16 from largest to smallest are 16, 8, 4, 2, 1.

The first factor that also divides 56 exactly is 8, therefore the GCF of 15 and 56 is 8.

1. Factor each number completely and find their common prime factors. The product of the common prime factors is the GCF of the numbers. Example—finding the GCF of 36 and 90:

Factors of 36: **2** x 2 x **3** x **3**

Factors of 90: **2** x **3** x **3** x 5

The common prime factors are 2, 3 and 3. Therefore, the GCF of 36 & 90 = 2 x 3 x 3 = 18. This method also works well for finding the GCF of 3 or more numbers.

**Super Cool GCF Trick (with special thanks to Euclid of Alexandria)**

Find the Greatest Common Factor of 391 and 713:

1. Divide the smaller of the two numbers into the larger and note the remainder

713/**391** = 1, with a remainder of **322**.

1. Divide the remainder into the smaller of the original 2 numbers and note the new remainder

391/**322** = 1, with a remainder of **69**

1. Divide the new remainder into the smaller of the two numbers in the preceding problem and note the new remainder

322/**69** = 4, with a remainder of **46**

1. Keep repeating this process until the remainder = 0

69/**46** = 1, remainder **23**

46/**23** = 2, remainder 0

1. The divisor (number you’re dividing by) that produced the 0 remainder is the GCF. The GCF of 391 and 713 is 23!

Least Common Multiple

The Least Common Multiple (LCM) of two numbers is the smallest number that is divisible by both numbers. Some strategies for finding the LCM:

1. List the multiples of each number, compare the lists and find the smallest common multiple. Example—find the LCM of 6 and 9:

Multiples of 6: 6, 12, **18**, 24, 30, **36**, 42…

Multiples of 9: 9, **18**, 27, **36**, 45, 54, 63…

The smallest number both lists have in common is 18, therefore the LCM of 6 & 9 is 18.

1. Test multiples of the larger number in order, starting with the number itself. Divide each of these multiples by the smaller number. The first multiple that the smaller number divides exactly is the LCM of the original two numbers. Example—find the LCM of 6 and 14:

Multiples of 14: 14, 28, **42**, 56, 70…

The first multiple that 6 divides exactly is 42, therefore the LCM of 6 & 14 is 42.

1. Factor each number completely. The LCM will be the product of the different prime factors, each taken the greater number of times it appears for either number. Example—find the LCM of 36 and 40:

Factors of 36: 2 x 2 x 3 x 3 (2 “2”s and **2 “3”s**)

Factors of 40: 2 x 2 x 2 x 5 (**3 “2”s** and **1 “5”**)

The LCM of 36 and 40 is 2 x 2 x 2 x 3 x 3 x 5 = 360. This method also works well for finding the LCM of 3 or more numbers.

1. Use the GCF of both numbers. The LCM of any two numbers is equal to the two numbers multiplied together, divided by their Greatest Common Factor. Example—find the LCM of 24 and 30:

The GCF of 24 & 30 is 6. (24 x 30)/6 = 120. Therefore, the LCM of 24 & 30 is 120.

1. Use a Venn diagram. Example—find the LCM of 30 and 42:

Factor the numbers completely: 30 = 2 x 3 x 5 and 42 = 2 x 3 x 7

The left oval is for the factors of 30; the right oval is for the factors of 42. The factors they have in common go in the middle section belonging to both ovals. The LCM is the product of the factors shown in the diagram. Therefore, the LCM of 30 & 42 = 5 x 2 x 3 x 7 = 210.

5

2, 3

7

**A Cool Way to find the LCM of Two or More Numbers**

1. Suppose you need to find the LCM of 15 and 18. Draw a chart and put the two numbers at the top next to a column labeled “Prime”.

|  |  |  |
| --- | --- | --- |
| **Prime** | **15** | **18** |
|  |  |  |

1. Next, find any prime that is a factor of any of the original numbers. List that prime on the first line under “Prime”.

|  |  |  |
| --- | --- | --- |
| **Prime** | **15** | **18** |
| 2 |  |  |

1. Look at the numbers that are on the previous row in the other columns. If the prime number listed does not divide the number exactly, repeat that number below it in the same column. If it does divide the number, do the division and put the quotient below it in the same column.

|  |  |  |
| --- | --- | --- |
| **Prime** | **15** | **18** |
| 2 | 15 | 9 |

1. Repeat the process. Find a prime that is a factor of any of the numbers in the lowest row. List that prime in the “Prime” column. Look at the numbers on the previous row. If the prime number does not divide the number exactly, repeat that number below it. If it does, do the division and put the quotient below it in the same column

|  |  |  |
| --- | --- | --- |
| **Prime** | **15** | **18** |
| 2  3  3  5 | 15  5  5  **1** | 9  3  1  **1** |

1. Continue to repeat this process until each of the numbers in the right-hand columns is a 1. The LCM of the original numbers will be the product of the primes listed in the table under “Primes”. Therefore, the LCM of 15 & 18 is 2 x 3 x 3 x 5 = 90.