Ratio & Proportion

Ratio

A ratio is a comparison of two numbers. We generally separate the two numbers in the ratio with a colon (:). Suppose we want to write the ratio of 8 and 12. We can write this as 8 : 12 or as a fraction $\frac{8}{12}$, and we say the ratio is *eight to twelve*. For example:

Suppose Sara has a bag with 3 DVDs, 4 marbles, 7 books, and 1 orange.

1. What is the ratio of books to marbles?

Expressed as a fraction, with the numerator equal to the first quantity and the denominator equal to the second, the answer would be $\frac{7}{4}$. Other ways of writing the ratio are 7 books to 4 marbles, 7 to 4, or 7 : 4.

1. What is the ratio of DVDs to the total number of items in the bag?

There are 3 videocassettes, and 3 + 4 + 7 + 1 = 15 items total.
The answer can be expressed as $\frac{3}{15}$, 3 to 15, or 3 : 15.

Comparing Ratios

To compare ratios, write them as fractions. The ratios are equal if they are equal when written as fractions. For example:

1. Are the ratios 3 to 4 and 6 : 8 equal?
2. The ratios are equal if $\frac{3}{4}$ = $\frac{6}{8}$

These are equal if their cross products are equal; that is, if 3 × 8 = 4 × 6. Since both of these products equal 24, the answer is yes, the ratios are equal.

Remember to be careful! Order matters!
A ratio of 1 : 7 is not the same as a ratio of 7 : 1. For example:

Are the ratios 7 : 1 and 4 : 81 equal? No!

$\frac{7}{1}$ > 1, but $\frac{4}{81}$ < 1, so the ratios can't be equal.

Are 7 : 14 and 36 : 72 equal? Yes!

Notice that $\frac{7}{14}$ and $\frac{36}{72}$ are both equal to $\frac{1}{2}$, so the two ratios are equal.

Proportion

A proportion is an equation with a ratio on each side. It is a statement that two ratios are equal.
$\frac{3}{4}$ = $\frac{6}{8}$ is an example of a proportion.

When one of the four numbers in a proportion is unknown, cross products may be used to find the unknown number. This is called solving the proportion. Question marks or letters are frequently used in place of the unknown number. Example:

Solve for n if $\frac{1}{2}$ = $\frac{n}{4}$.

Using cross products we see that 2 × n = 1 × 4 = 4, so 2 × n = 4.

Dividing both sides by 2, n = 4 ÷ 2, therefore n = 2.

Continued Ratios

Continued ratios are used to compare more than two numbers together. They are also usually written with colons (:). For example, the ratio of 4 to 8 to 12 is written 4 : 8 : 12. Example:

Thomas needs 2 spiral notebooks, 4 pencils, 1 red correcting pen and 2 sharpies for each of his classes every day. If he has 45 total items in his bag, how many classes does he take?

The continued ratio is 2 notebooks to 4 pencils to 1 pen to 2 sharpies, or 2 : 4 : 1 : 2. We’ll use “x” to represent the number of classes he takes. So the ratio can be rewritten as:

(2 notebooks times the number of classes) + (4 pencils times the number of classes) +

(1 pen times the number of classes) + (2 sharpies times the number of classes) = 45 total items

or

2x + 4x + 1x + 2x = 45

Solving for x, 9x = 45, therefore x = 5. Thomas is taking 5 classes.

Quick Reference – Fractions

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| **Cancelling** | **Cancelling is the act of removing common factors from the numerator and denominator of a fraction.**  | $\frac{9}{36}$ **=** $\frac{1 x 3 x 3}{1 x 2 x 2 x 3 x 3}$ **=** $\frac{1}{4}$ |
| **Common Denominator** | **If two or more fractions have the same number for a denominator, they are said to have a common denominator.** | $\frac{1}{12}$ **,** $\frac{5}{12}$ **and** $\frac{11}{12}$**have a common denominator (12)** |
| **Denominator** | **The number which tells how many equal parts are needed to make a whole (the bottom number on a fraction)** | **The denominator** **of** $\frac{1}{4}$ **is 4** |
| **Equivalent Fractions** | **Fractions that represent the same number** | $\frac{9}{36}$ **and** $\frac{1}{4}$ **are equivalent fractions because** $\frac{9}{36}$ **=** $\frac{1}{4}$ |
| **Improper Fraction** | **A fraction where the denominator is smaller than the numerator** | $\frac{4}{3}$ **is an improper fraction** |
| **Lowest Terms** | **When the greatest common factor (GCF) of both the numerator and denominator of a fraction is 1, the fraction is in its lowest terms** | $\frac{9}{36}$ **reduced to its lowest terms is** $\frac{1}{4}$ |
| **Mixed Number** | **When a whole number is combined with a fraction** | **1** $\frac{1}{3}$ **is a** **mixed number** |
| **Numerator** | **The number that tells how many parts of the whole are in a fraction (the top number on a fraction)** | **The numerator** **of** $\frac{1}{4}$ **is 1** |
| **Proper Fraction** | **A fraction where the numerator is smaller than the denominator** | $\frac{3}{4}$ **is a proper fraction** |
| **Reciprocal** | **The inverse of a fraction, or what you get if you flip a fraction upside down** | **The reciprocal** **of** $\frac{3}{4}$ **is** $\frac{4}{3}$ |

Fractions

Adding Fractions

To add fractions with a common denominator, simply add their numerators together and write the sum over their shared denominator. Example:

$\frac{3}{10}$ + $\frac{4}{10}$ **=**  $\frac{3 + 4}{10}$ **=**  $\frac{7}{10}$

Fractions having unlike denominators must first be converted into fractions having a common denominator. First, find the lowest common multiple (LCM) of both fractions’ denominators. Next, multiply each fraction’s numerator by the same number used to multiply its denominator to equal the LCM. Once both fractions are given a common denominator in this way, the addition can be performed. Example: $\frac{1}{4}$ + $\frac{1}{6}$ . The LCM of 4 & 6 is 12.

$\frac{1}{4}$ + $\frac{1}{6}$ **=**  $\frac{3}{12}$ + $\frac{2}{12}$ **=**  $\frac{3 + 2}{12}$ **=**  $\frac{5}{12}$

Subtracting Fractions

Subtracting fractions is done the same way as adding fractions, finding common denominators as needed, except the numerators are subtracted instead of added. Examples:

$\frac{7}{10}$ - $\frac{4}{10}$ **=**  $\frac{7- 4}{10}$ **=**  $\frac{3}{10}$ $\frac{1}{4}$ - $\frac{1}{6}$ **=**  $\frac{3}{12}$ - $\frac{2}{12}$ **=**  $\frac{3 - 2}{12}$ **=**  $\frac{1}{12}$

Simplifying Fractions

Fractions should always be reduced to their lowest terms. To do this, simply factor out (eliminate) all common factors shared by the numerator and denominator. Example:

$\frac{32}{72}$ **=**  $\frac{2 x 2 x 2 x 2 x 2}{2 x 2 x 2 x 3 x 3}$ **=**  $\frac{2 x 2}{3 x 3}$ **=** $\frac{4}{9}$

Converting Improper Fractions to Mixed Numbers

 To convert an improper fraction to a mixed number, divide the numerator by the denominator to find the whole number, and then place the remainder of the division problem over the denominator to find the remaining fraction. Example:

$\frac{23}{5}$ **=** 23 $÷$ 5 **=** 4 with a remainder of 3 **=** 4 $\frac{3}{5}$

Converting Mixed Numbers to Improper Fractions

To convert a mixed number to an improper fraction, multiply the denominator by the whole number, then add the numerator. Place this total over the denominator. Example:

1 $\frac{1}{3}$ **=** $\frac{\left(3 x 1\right)+1}{3}$ **=** $\frac{4}{3}$

Multiplying Fractions

In multiplication the numerators of the fractions are multiplied together as are the denominators. Convert mixed numbers to improper fractions before multiplying, and simplify your result. Example:

1 $\frac{1}{3}$ x $\frac{1}{8}$ **=** $\frac{4}{3}$ x $\frac{1}{8}$ **=** $\frac{4}{24}$ = $\frac{1}{6}$

Dividing Fractions

The division of one fraction by another is performed by inverting the divisor (the number the first number is being divided by) and multiplying. Example:

$\frac{3}{5}$ $÷$ $\frac{1}{2}$ **=** $\frac{3}{5}$ x $\frac{2}{1}$ **=** $\frac{6}{5}$ **=** 1 $\frac{1}{6}$