1. How many pairs of integers (whole numbers) have a product of 400?

2. Ms. Pearl is shopping at a supermarket to prepare for next Sunday's picnic. She sees that hamburger buns come in packages of 8 but the hamburger patties are in packages of 10. What is the least number of packages she should get of each so that there are no leftover buns or patties?

3. Mr. Brackett works in factory with his two sons. He is really happy with his job because he gets to spend some time with his sons at work! Mr. Brackett is allowed to take a break every 140 minutes. His older son takes a break every 210 minutes, and his younger son takes a break every 280 minutes. After their first break together, how many minutes will they have to wait until they all have a break together again?

4. Mr. and Mrs. White are soon moving into their new home, and would like to have their bathroom floor tiled. The bathroom is rectangular with sides 460cm and 600cm. What's the length of the side of the biggest square-shaped tile (in centimeters) that could be used so that no space is left without tiles? (Assume tiles can't be cut, and there is no gap between adjacent tiles.)

5. When people joining a competition are grouped in 4's, 5's or 6's, 1 person is left out each time. If we know that there are at least 150 people joining the competition, what is least number of people in the competition?

6. The number 45N78 is divisible by 3. What are all the possible values of N?

7. Sophia, a bookworm, loved books so much that she filled her bookcase with 40 books on each shelf. After noticing that the shelves were starting to bend, she decided to buy another smaller bookcase that had 4 shelves. When she distributed all of her books equally between all of the shelves on the two bookcases, there were 24 books on each shelf. How many shelves were on her original bookcase?

8. What is the smallest positive integer that is divisible by each of the numbers 1 through 7?

BONUS PROBLEMS

9. Given a four digit number, 55NN, for what value(s) of N will this four digit number be divisible by 8 and 9?

10. Guess this number from the clues:
    a. The least common multiple of this number and 42 is 630.
    b. The greatest common factor of this number and 42 is 6.

11. What is the sum of the prime factors of 2002?

12. How many positive integers less than 1000 are divisible by 4 and 6, but not 8?

Solutions:

1. **8 pairs**
   (1, 400), (2, 200), (4, 100), (5, 80), (8, 50), (10, 40), (16, 25), (20, 20)

2. **5 packages of buns and 4 packages of patties**
   We use the LCM (least common multiple). LCM(8,10) = 40. This means she would have to make at least 40 hamburgers so that nothing is left behind. So she needs 40/8 = 5 packages of buns and 40/10 = 4 packages of burgers.

3. **840 minutes**
   Use the LCM (least common multiple) to find the answer. LCM(140,210,280) = 840. In 840 minutes, the father will get to see his sons again. (I know this is quite a long shift but let's assume Mr.Brackett and sons are really, really hard workers!)

4. **20cm**
   use the GCF (greatest common factor). GCF(460,600) = 20cm The square tiles should each have a side of 20cm at most.

5. **181 people**
   First let's find the LCM. LCM(4,5,6) = 60. The answer would have been 60+1 = 61 if we were not told that there are more than 150 people. Since the number still has to be a multiple of 60 (plus 1) above 150, the answer is 181.

6. **0, 3, 6, and 9**
   The divisibility rule for 3 says that the sum of the digits must be divisible by 3. Since 4+5+N+7+8 must be divisible by 3, possible values for N are 0, 3, 6 & 9.

7. **6 shelves**.
   Since she originally had 40 books on each shelf, the total number of books must be a multiple of 40. Since she ended up with 24 books on each shelf, the total number of books must also be a multiple of 24. So the total number of books could be any number that is a multiple of both 24 and 40, such as 120, 240, 360, etc..

   Checking to see if 120 works in the problem, we see that it does not. She would have had to have 3 shelves to start with (3 x 40 = 120), and 5 total shelves in the end (5 x 24 = 120). But we know she added 4 shelves, not 2.

   Checking to see if 240 works, we see that it does. She had 6 shelves to start with (6 x 40 = 240), and 10 total shelves in the end (10 x 24 = 240). This works, because we know she got four new shelves.

8. **420**
   Taking the prime factors 1 x 2 x 3 x (2 x 2) x 5 x (2 x 3) x 7, you can cancel out one of the 2s from 2 x 2 (4) and both the 2 & the 3 from 2 x 3 (6), which leaves 1 x 2 x 3 x 2 x 5 x 7 = 420

9. **N=4**
   To be divisible by 9, the sum of all the digits must be divisible by 9 (5+5+4+4 = 18). N=4 is the only value that makes the number divisible by 9. And this also happens to work for divisibility by 8.

10. **90**
    Prime factors of 42 are 2, 3 & 7. Prime factors of 630 are 2, 3, 3, 5 & 7. Therefore, the number must have 2 & 3 as prime factors (for the GCF of 6), plus an additional 3 & 5 as prime factors (factors of 630 that are in addition to the factors of 42), giving us 2 x 3 x 3 x 5 = 90.

11. **33**
    2 x 7 x 11 x 13 = 2002
    2 + 7 + 11 + 13 = 33

12. **42**
    The LCM of 4 and 6 is 12. So, the numbers that are divisible by 4 and 6 are:
    12, 24, 36, 48, 60, 72, 84, etc.
    But every other one of these numbers is divisible by 8, so the only numbers that satisfy the entire condition are:
    12, 36, 60, 84, etc.
    Following this pattern all the way to 1000, we find there are 42 numbers that qualify.