1. What is the sum of all the prime numbers between 50 and 60?

2. In lowest terms, what is the product of this multiplication expression?
$$\frac{3}{5}\times\frac{5}{7}\times\frac{7}{9}\times\frac{9}{11}\times\frac{11}{13}\times\frac{13}{15}\times\frac{15}{17}\times\frac{17}{19}\times\frac{19}{21}\times\frac{21}{23}\times\frac{23}{25}\times\frac{25}{27}$$

3. Find the whole number value of n:
$$\frac{3}{80} < \frac{1}{n} < \frac{4}{101}$$

4. Mary and her brother Max had a contest to see who could swim the farthest in a short amount of time. They found that Mary could swim twice as far as Max. If the total distance they swam was 120 yards, how many yards was Mary able to swim?

5. Each row of *s has two more *s than the row immediately above it, as shown. Altogether, how many *s are contained in the first twenty rows?

   *

   * * *

   * * * * *

   * * * * * * *

   and so on…

6. In a "Tribonacci Sequence," each number after the third number is the sum of the preceding three numbers. For example, if the first three numbers are 5, 6, and 7, then the fourth number is 18 because 5 + 6 + 7 = 18 and the fifth number is 31 because 6 + 7 + 18 = 31. The first five numbers of another Tribonacci Sequence are P, Q, 86, 158, and 291 in that order. What is the value of P?

7. Michelle's Number Recycling Machine obeys exactly two rules:
   - If an inserted number has exactly 1 digit, double the number.
   - If an inserted number has exactly 2 digits, compute the sum of the digits.

   The first number Michelle inserts is 1. Then every answer she gets is inserted back into the machine until fifty numbers are inserted. What is the fiftieth number to be inserted?

8. Bryan can buy candy canes at 4 for 50¢ and can sell them at 3 for 50¢. How many canes must Bryan sell in order to make a profit of $5.00?

BONUS PROBLEMS

9. Assume that a post office issues only 3¢ and 8¢ stamps and all postage is in whole numbers of cents. What is the greatest amount of postage in cents which *cannot* be made using only 3¢ and 8¢ stamps?

10. It takes Melissa 2 hours to paint one car. It takes Dave 4 hours to paint one car. Kate also takes 4 hours to paint one car. If they all work together, how long will it take them to paint 2 cars?

11. In simplest terms, what is the value of the following?

$$\frac{6}{.3} + \frac{.3}{.06}$$

12. Find the 4-digit number ABCD if:

$$\begin{array}{r} A\ B\ C\ D \\ \times \quad\ 9 \\ \hline D\ C\ B\ A \end{array}$$

SOLUTIONS:

1. **112**
   Use the tests of divisibility to find the prime numbers between 50 and 60. First, 52, 54, 56, and 58 are multiples of 2. Next, 51, 54, and 57 are multiples of 3 because the sum of the digits of each is also divisible by 3. Then, 55 is a multiple of 5. That leaves 53 and 59, both of which are prime. The sum of the prime numbers between 50 and 60 is 112.

2. $\frac{1}{9}$
   Cancel identical numerators and denominators with each other (that is, divide out each common factor greater than 1). This can be done 11 times. Then we are left with $\frac{3}{27}$. In lowest terms, the product is $\frac{1}{9}$.

3. **26**
   Estimate the given fractions using simpler fractions. $\frac{3}{81}$, which equals $\frac{1}{27}$, is less than $\frac{3}{80}$. $\frac{4}{100}$, which equals $\frac{1}{25}$, is greater than $\frac{4}{101}$. Thus, $\frac{1}{n}$ is between $\frac{1}{25}$ and $\frac{1}{27}$, and the value of n is $\frac{1}{26}$.

4. **80 yards**
   Since Mary swims twice as fast, for every distance Max swims, Mary swims twice as far. So in this case, x (Max's distance) + 2x (Mary's distance) = 120, 3x = 120, x = 40, so Mary's distance = 2x = 80.

5. **400**
   Set up a table and look for a pattern.

Number of Rows	1	2	3	4	5	…	20
Total Number of Stars	1	4	9	16	25	…	?

   The total number of stars by the end of each row is the square of the number of rows to that point. By this pattern, the first 20 rows will have a total of 20 x 20 = 400 stars. 400 stars are contained in the first 20 rows.

6. **25**
   Work backwards. First Q + 86 + 148 = 291, so Q = 47.
   Then P + 47 + 86 = 158, so P = 25.

7. **16**
   Use the rules to generate the first numbers until a pattern appears.

   1, 2, 4, 8, 16, 7, 14, 5, 10, 1…

   Since the first and tenth numbers are the same, the machine repeats itself every 9 numbers. Thus, the 10^{th}, 19^{th}, 28^{th}, 37^{th}, and 46^{th} numbers each are 1. Similarly, the 5^{th} and 50^{th} numbers are the same number. The 50^{th} number is 16.

8. 120

Assume he buys and sells 12 canes in order to avoid using fractions.
12 canes cost Bryan 3 x $.50 = $1.50
12 canes sell for 4 x $.50 = $2.00
12 canes bring a profit of $2.00 - $1.50 = $.50
Since $5.00 = $.50 x 10, multiply 12 canes by 10 = 120 canes.

9. 13

All whole numbers leave a remainder of 0, 1, or 2 after division by 3, so we can divide them into three independent sets of numbers according to their remainders:

Remainder 0: All multiples of 3 can be made using only 3¢ stamps: 3, 6, 9, 12, 15,…
Remainder 2: Use one 8¢ stamp and any number of 3¢ stamps. Beginning with 8¢, 2¢ more than all multiples of 3 can be made: 8, 11, 14, 17, etc. Amounts that cannot be made are 2 and 5.
Remainder 1: Use two 8¢ stamps and any number of 3¢ stamps. Beginning with 16¢, 1¢ more than all multiples of 3 can be made: 16, 19, 22, etc. Amounts that cannot be made are 1, 4, 7, 10 and 13.

Since amounts of 14¢, 15¢ and 16¢ can be made, every whole number that follows can be made by merely adding 3¢ stamps. Thus, the greatest amount of postage that can't be made is 13¢.

10. **2 hours**

Dave paints ¼ of a car in an hour. Kate paints ¼ of a car in an hour. Melissa paints ½ of a car in an hour. The three fractions add up to one whole car; therefore it takes 2 hours to paint two cars.

11. **25**

The meaning of any fraction $\frac{a}{b}$ is a ÷ b. Therefore, $\frac{6}{.3}$ means 6 ÷ 0.3, whose value is 20. Similarly, $\frac{3}{.06}$ means 0.3 ÷ .06, whose value is 5. The value of the sum is 25.

12. 1089

A is 1, since any greater value produces a 5-digit product when multiplied by 9. D is 9, since only 9 times 9 results in a 1 in the ones place. B is 0, since 9 times any other number (including 1) results in a "carry" into the thousands place. C is 8, since 9 times C ends in 2, so that when the 8 "carry" is added, 0 is produced in the tens place.